Miss Distance Analysis for Command Guided Missiles

Joel Alpert
Massachusetts Institute of Technology, Lincoln Laboratory, Lexington, Massachusetts

A concise theoretical technique is presented for estimating the minimum miss distance capability of command guided missile systems using synthetic proportional navigation. The effect of the parameter values on the system capability is shown to be a function of range-to-intercept; the technique enables the system designer and analyst to quantify system performance and to develop a systematic understanding of the performance limitations of command guidance systems at each intercept range. New analytical equations based upon adjoint theory are developed for statistical miss distance caused by target maneuver, range-dependent, servo, glint and atmosphere noises for command guided systems. An optimal total system time constant is derived which yields the minimum statistical miss distance. Realistic constraints on the minimum achievable system time constant are considered. The equations derived for the optimal total system time constant are valuable to the system designer for minimizing miss distance over the ranges of system parameters and limitations, and intercept conditions.

I. Introduction

COMMAND guidance systems can be less expensive than homing guidance systems. Nevertheless, few command guidance systems are currently being built, because it is believed that command guidance miss distances are too large for successful interceptions. In order to identify the conditions where the less expensive command guidance system accuracy may be satisfactory, a systematic analysis of command guidance miss distance would be most useful. This paper presents such an analysis.

In command guided missile systems, a ground radar tracks both the target and the missile, computes guidance commands, and uplinks them to the missile. The missile accuracy is limited by the noises in the systems. The effect of these noises for command guidance is known to be strongly dependent upon the range to the object tracked; the dependence on range-to-intercept is much weaker for either active or semiactive homing guidance. Previous work[1] explores command guidance system performance at specific ranges-to-intercept. No systematic analysis of the dependence on range-to-intercept of command guidance system performance exists. Statistical miss distance performance has been solved for the homing missile system using adjoint theory,[2] but not for command guidance missile systems.

In this paper, closed-form steady-state adjoint solutions for statistical miss distance are derived for command guidance systems that use proportional navigation. The major contributions of this paper are the derivation of the optimal total system time constant that achieves the minimum statistical miss distance and the development of a technique that clearly shows the dependence of command guidance system performance upon the intercept range and system parameters. The technique indicates the system's performance limitations based upon the interplay of the various system parameters. It also elucidates the ranges-to-intercept at which the various noises dominate and hence determine the minimum achievable statistical miss distance.

II. Steady-State Adjoint Solutions for Command Guidance

In this section, the steady-state adjoint solutions are developed for the rms miss distance due to various noises. They are derived by applying the adjoint method to command guided missiles that use synthetic proportional navigation. Synthetic

Joel Alpert received B.S. and M.S. degrees in electrical engineering from the University of Wisconsin in 1967 and 1968, respectively. Since 1982 he has been on the research staff at MIT Lincoln Laboratory. He has been analyzing missile interceptor system performance with both analytical techniques and six-degree-of-freedom simulations, and has been involved with advanced seeker concepts. Before 1982 he worked at Raytheon Missile Systems Division, primarily on the guidance system design for a new air-to-air missile. Mr. Alpert also worked at Bell Telephone Laboratories on the Safeguard ABM system.

Presented as Paper 87-2582 at the AIAA Guidance, Navigation, and Control Conference, Monterey, CA, Aug. 19–21, 1987; received June 12, 1987; revision received Dec. 22, 1987.

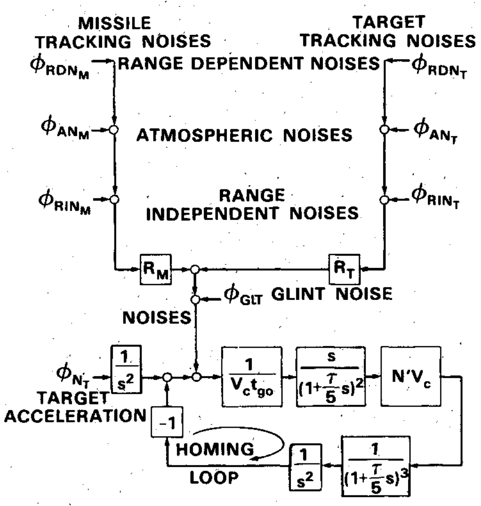

Fig. 1 Command guidance system diagram with noises.

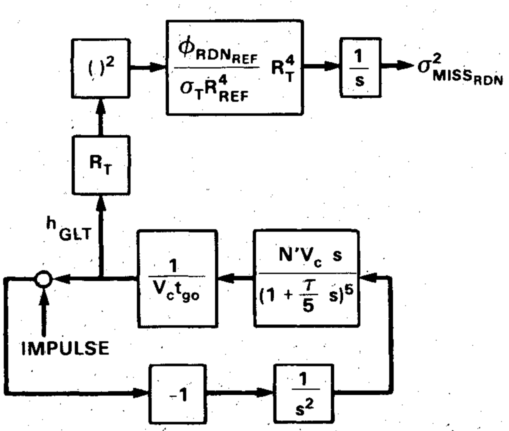

Fig. 2 Adjoint diagram of command guidance system with range-dependent noise.

proportional navigation is defined to be standard proportional navigation guidance except that the required line-of-sight from the missile to the target is mathematically constructed based upon fire control radar measurements; consequently it contains the radar's measurement errors of the missile and target. It is assumed in this analysis that the ranges and closing velocity are known perfectly. Figure 1 is the standard linearized system block diagram of the missile intercept problem that uses synthetic proportional navigation in the command guidance mode.[1] It contains the standard homing loop for a missile using proportional navigation and the fire control radar noises present in a command guidance system. The system is assumed to be fifth order, which is considered the minimum order that realistically represents a missile.[2] The signal (target acceleration) and noise levels are represented by their respective spectral densities, ϕ. All the noises are input at the point just before the division by the range between the missile and the target, which is closing velocity times time-to-go ($V_c t_{go}$). The fire control radar tracks the missile and the target at a rate of f_s, which is assumed to be high enough so that the track loop can be considered continuous, not sampled-data. Note that all fire control radar noises, which are in angular units, are multiplied by the range from the radar to the object being tracked (i.e., R_T for the target and R_M for the missile), before injection into the homing loop. Table 1 contains further explanation of the signal and noise parameters used in Fig. 1.

The steady-state adjoint solution for miss distance is developed here only for the range-dependent noise, because it is the most complex expression of all of the noises; the steady-state adjoint solutions for miss distance caused by the other noises are simply presented, because their development is similar.

Table 1 Command guidance noise models

Range-dependent noise	
$\phi_{RDN} = \dfrac{\phi_{RDN\,REF}}{\sigma_T}\left(\dfrac{R}{R_{REF}}\right)^{4a}$	where
$\phi_{RDN\,REF} = \dfrac{\sigma^2_{RDN\,REF}}{f_s}$	$R = R_M$ or R_T BW = beamwidth in radians σ_T = radar-cross-section in m^2
$\sigma_{RDN\,REF} = \dfrac{BW}{1.6(S/N_{REF})^{1/2}}$	$S/N_{REF} = S/N$ for 1 m^2 target at Reference Range R_{REF} where f_s is the data rate in Hz
Range-independent noise	
$\phi_{RIN} = \dfrac{\sigma^2_{RIN}}{f_s}$	where $\sigma_{RIN} = BW/BSR$ and BSR is the beam splitting ratio for large S/N
Glint noise	
$\phi_{GLT} = 2\tau_{GLT}\sigma^2_{GLT}$	where $\sigma_{GLT} = W_s/5^b$ and W_s = wing span of target where τ_{GLT} = glint noise correlation time
Atmospheric noise[1,10]	where
$\phi_{AN} = \phi_{AN\,REF}R^*$	R^* = path length in m to tracked object in lower 5 km of atmosphere
$\phi_{AN\,REF} = \dfrac{4\tau_{AN}\sigma^2_{AN\,REF}f_c}{w^{1/2}}$	w = antenna aperture in m $\sigma_{AN\,REF} = .44 \times 10^{-6}$ rad τ_{AN} = atmospheric noise correlation time = 0.6 s f_c = correlated noise coefficient = 0.4, see section on correlated noise effects
Target of maneuver	
$\phi_{MVR} = N_T^2/T_M$ for a step target maneuver of N_T m/s^2	where T_M is the maneuver time. For a Poisson distributed starting time for the step maneuver with average time between maneuvers of T_P, $T_M = 0.25\ T_P$.

[a]If the missile carries a beacon, then $\phi_{RDN} = \phi_{RDN_{REF}}(R/R_{REF})^2$ where $\phi_{RDN_{REF}}$ and R_{REF} are defined for the beacon.
[b]Factor of 5 is a rule of thumb. It can vary between 2 and 8 according to data.

Figure 2 is the adjoint system representation of the system with range-dependent noise from the target track only.

The method of adjoints is described in the literature.[3-7] Reference 3 contains the technique for developing the adjoint system diagram, Fig. 2, from the linearized system diagram (often called the forward system diagram), Fig. 1.

Note that there is a multiplying factor of R_T^6 for the track of the target (or R_M^6 for the track of the missile) between the point where the signal leaves the adjoint of the homing loop as h_{GLT} and the place where it enters the integrator. For the target

$$R_T = R_I + V_T t_{go} \tag{1}$$

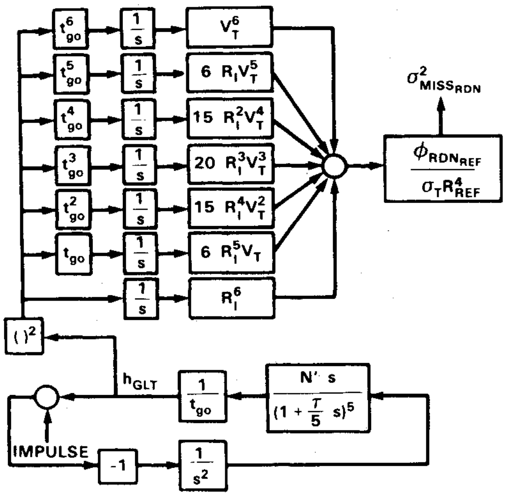

Fig. 3 Equivalent adjoint diagram of command guidance system with range-dependent noise.

where

R_I = intercept range

V_T = target velocity (positive for incoming target)

t_{go} = time to intercept

Therefore

$$R_T^6 = \sum_{n=0}^{6} \binom{6}{n} R_I^{6-n} V_T^n t_{go}^n \tag{2}$$

Using Eq. (2), Fig. 3 can be developed from Fig. 2.

It can be shown that the output of the integrator with a t_{go}^n factor in the integrand reaches a steady-state value at t_{ss}

$$K_n \tau^{n-1}$$

where K_n's are the normalized adjoint coefficients and τ is the sum of system time constants.

$$K_n = \tau \int_{t^*=0}^{t_{ss}} \left(\frac{t^*}{\tau}\right)^n h_{GLT}^2(t^*)\, dt^* \tag{3}$$

$$t^* = t_{go} \text{ is adjoint time}$$

Consequently, the steady-state adjoint mean square (ms) miss distance due to range-dependent is:

$$\sigma^2_{MISS_{RDN}} = \frac{\phi_{RDN_{REF}}}{R^4_{REF}\sigma_T} \left[\sum_{n=0}^{6} \binom{6}{n} K_n R_I^{6-n} V_T^n \tau^{n-1} \right] \tag{4}$$

Using this same procedure, the miss distances for atmospheric noise and range-independent noise (often referred to as fading noise or servo noise) can also be developed. While range-dependent noise miss distance variance is a sixth-order polynomial of intercept range, the corresponding atmospheric noise miss distance variance is a third-order polynomial, and that for range-independent noise a second order.

The steady-state adjoint solution for ms miss distance caused by target maneuver is also of interest and can be shown to be

$$\sigma^2_{MISS_{MVR}} = \phi_{MVR} K_M \tau^5 \tag{5}$$

where

$$K_M = \frac{1}{\tau^5} \int_{t^*=0}^{t_{ss}} \left[\int_0^{t_{ss}} \int_0^{t_{ss}} \left(\int_0^{t_{ss}} h_{GLT}(t^*)\, dt^* + 1 \right) dt^*\, dt^* \right]^2 dt^* \tag{6}$$

Table 2 Normalized steady-state miss distance adjoint coefficients

	$N' = 3$	$N' = 3.5$	$N' = 4$
K_0	2.8	3.97	5.5
K_1	4.4	6.7	10.0
K_2	9.2	15.6	25.7
K_3	25.9	47.6	84.8
K_4	89.7	176.0	336.0
K_5	365.0	742.0	1541.0
K_6	1707.0	3750.0	7979.0
K_M	3.77	2.38	1.98

Table 3 Steady-state adjoint miss distance equations

Command guidance for target track[a]

$$\sigma^2_{MISS_{RDN}} = \frac{\phi_{RDN_{REF}}}{\sigma_T R^4_{REF}} \sum_{n=0}^{6} \binom{6}{n} K_n R_I^{6-n} V_T^n \tau^{n-1}$$

$$\sigma^2_{MISS_{AN}} = \phi_{AN_{REF}} \sum_{n=0}^{3} \binom{3}{n} K_n R_I^{3-n} V_T^n \tau^{n-1}$$

$$\sigma^2_{MISS_{RIN}} = \phi_{RIN} \sum_{n=0}^{2} \binom{2}{n} K_n R_I^{2-n} V_T^n \tau^{n-1}$$

$$\sigma^2_{MISS_{GLT}} = \phi_{GLT} K_o \tau^{-1}$$

$$\sigma^2_{MISS_{MVR}} = \phi_{MVR} K_M \tau^5$$

Homing guidance[b]

$$\sigma^2_{MISS_{RDN}} = \frac{\phi_{RDN_{REF}}}{\sigma_T R_P^4} (V_c^4 \tau^4) \sum_{n=0}^{2} \binom{2}{n} K_{n+4} R_I^{2-n} V_T^n \tau^{n-1}$$

(Semiactive)

$$\sigma^2_{MISS_{RIN}} = \phi_{RIN} K_2 V_c^2 \tau$$

$$\sigma^2_{MISS_{GLT}} = \phi_{GLT} K_o \tau^{-1}$$

$$\sigma^2_{MISS_{MVR}} = \phi_{MVR} K_M \tau^5$$

[a]For missile track V_M replaces V_T.
[b]$R_P^2 = R_{IT_{REF}} R_{MT_{REF}}$ the product of the illuminator-to-target range and missile-to-target range at which there is S/N_{REF} on a 1 m^2 target.

It turns out that the normalized adjoint coefficients, K_n, are dimensionless and independent of the system time constant, τ, and closing velocity, V_c, and are functions only of the order of the system (fifth, in this case) and the navigation ratio, N'. Hence, given N' and the order of the system, and the distribution of the time constraints,[2] these coefficients can be developed numerically once and for all. They can also be developed using the methodology of the appendix of Ref. 2.

Table 2 contains the values of normalized adjoint coefficients K_0 through K_6 and K_M for $N' = 3.0$, 3.5, and 4.0.

The steady-state adjoint equations for the mean square command guidance miss distance due to each noise source are given in Table 3 along with that for target maneuver. Also given in Table 3 are the corresponding equations for the semiactive homing guidance case.

III. Time Constant Optimization for Minimum Miss Distance

In this section, the optimal time constant that achieves the minimum stochastic miss distance is derived.

From this point on, it is assumed that the missile carries a beacon so that there is neither range-dependent noise nor glint noise on the missile track; there remain, however, range-independent noise and atmospheric noise.

The variance of the total miss distance is the sum of the variances of the miss distances due to each noise source and due to target maneuver:

$$\sigma^2_{\text{MISS}} = \left\{\left[\frac{\phi_{RDN_{\text{REF}}}}{\sigma_t R^4_{\text{REF}}} R_I^4 \left[\sum_{n=0}^{6} \binom{6}{n} \frac{K_n}{K_o} V_T^n \left(\frac{\tau}{R_I}\right)^n\right]\right.\right.$$
$$+ \phi_{RIN}\left[\sum_{n=0}^{2} \binom{2}{n} \frac{K_n}{K_o} (V_T^n + V_M^n) \left(\frac{\tau}{R_I}\right)^n\right]$$
$$\left.+ \phi_{AN_{\text{REF}}} R_I \left[\sum_{n=0}^{3} \binom{3}{n} \frac{K_n}{K_o} (V_T^n + V_M^n) \left(\frac{\tau}{R_I}\right)^n\right]\right]$$
$$\left. \times R_I^2 + \phi_{GLT}\right\} K_o \tau^{-1} + \phi_{MVR} K_M \tau^5 \qquad (7)$$

It can be shown that the first terms in the polynomial expressions for miss distance variance due to range-dependent noise, range-independent noise, and atmospheric noise are the largest terms for many practical values of target and missile velocity and missile system time constant. Therefore, Eq. (7) can be approximated by

$$\sigma^2_{\text{MISS}} = \left(\sum \phi\right) K_o \tau^{-1} + \phi_{MVR} K_M \tau^5 \qquad (8)$$

where

$$\sum \phi \approx \left(\frac{\phi_{RDN_{\text{REF}}}}{\sigma_T R^4_{\text{REF}}} R_I^4 + 2\phi_{RIN} + 2\phi_{AN_{\text{REF}}} R_I\right) R_I^2 + \phi_{GLT} \qquad (9)$$

Equation (9) can be identified as the sum of spectral noise densities at the intercept range.

Equation (8) shows that the variance of the miss due to the noises is approximately inversely proportional to the system time constant, and the variance of the miss due to target maneuver is proportional to the fifth power of the system time constant.

The optimal time constant for minimum rms miss distance can be found by taking the partial derivative of the expression with respect to τ, setting it to zero, and solving, thereby giving

$$\tau_{\text{OPT}} = 0.2\left(\frac{K_o \sum \phi}{K_M \phi_{MVR}}\right)^{1/6} \qquad (10)$$

for the optimal time constant, and for minimum rms miss distance:

$$\sigma_{\text{MISS}_{\text{MIN}}} = 1.25\left[K_M \phi_{MVR}\left(K_o \sum \phi\right)^5\right]^{1/12} \qquad (11)$$

For this optimal condition, maneuver miss distance accounts for 41% of the total rms miss distances.

IV. System Performance Asymptotes

In this section, a set of equations is developed to describe asymptotic miss distance performance. Each equation represents the minimum rms miss distance which could be achieved if only one class of guidance noise were present. Studying these equations gives the analyst an understanding of the effects that contribute to miss distance.

Sources of guidance noise influence minimum rms miss distance through the sum defined by Eq. (9). It is useful to recognize that the relative magnitudes of the terms in Eq. (9) depend explicitly upon intercept range and that, at any particular intercept range, one noise term dominates the others. Asymptotic miss distance is defined as the minimum rms miss distance which would be expected if only the dominant noise term were considered along with the maneuver term.

At the shortest intercept range, glint dominates and miss distance is independent of intercept range; the notation (GLT) indicates that the glint noise dominates.

$$\sigma_{\text{MISS}_{\text{MIN}(GLT)}} = 1.25[K_M \phi_{MVR}(K_o \phi_{GLT})^5]^{1/12} \qquad (12)$$

As intercept range is increased, the range-independent noise miss distance eventually increases to be equal to that of glint noise miss distance, at range

$$R_{I_1} = (\phi_{GLT}/2\phi_{RIN})^{1/2} \qquad (13)$$

Beyond this range the asymptotic miss is:

$$\sigma_{\text{MISS}_{\text{MIN}(RIN)}} = 1.25[K_M \phi_{MVR}(2K_o \phi_{RIN})^5]^{1/12} R_I^{5/6} \qquad (14)$$

In this region, where the range-independent noise dominates, the asymptotic miss distance is proportional to the five-sixths power of intercept range.

In the next region, where the intercept range is larger than

$$R_{I_2} = \phi_{RIN}/\phi_{AN_{\text{REF}}} \qquad (15)$$

the atmospheric noise dominates and the asymptotic miss distance is

$$\sigma_{\text{MISS}_{\text{MIN}(AN)}} = 1.25[K_M \phi_{MVR}(2K_o \phi_{AN_{\text{REF}}})^5]^{1/12} R_I^{5/4} \qquad (16)$$

The miss distance is proportional to the five-fourths power of intercept range.

The range-dependent noise dominates at intercept ranges larger than

$$R_{I_3} = \left(\frac{2\sigma_T \phi_{AN_{\text{REF}}} R^4_{\text{REF}}}{\phi_{RDN_{\text{REF}}}}\right)^{1/3} \qquad (17)$$

The asymptotic miss distance is

$$\sigma_{\text{MISS}_{\text{MIN}(RDN)}} = 1.25\left[K_M \phi_{MVR}\left(K_o \frac{\phi_{RDN_{\text{REF}}}}{R^4_{\text{REF}} \sigma_T}\right)^5\right]^{1/12} R_I^{5/2} \qquad (18)$$

The miss distance is proportional to intercept range to the five-halves power.

Asymptotic miss distance can be quickly drawn on log-log paper, once the various system parameters have been given values, the critical range points of Eqs. (13), (15), and (17) evaluated, and the miss distances of Eqs. (12), (14), (16), and (18) calculated. Only the slope of the miss distance curve with respect to intercept range changes between the regions.

Table 4 Parameter values used in the examples

BW	= 0.035 rad (2 deg)	Beamwidth
S/N_{REF}	= 100 (20 dB)	Reference S/N at reference range R_{REF}
R_{REF}	= 50,000 m	Reference range
σ_T	= 100, 10, 1, 0.1 m^2	Radar-cross-section
f_s	= 40 Hz	Data rate
BSR	= 80	Beam split ratio for large S/N
W_s	= 10 m	Wing span of target
τ_{GLT}	= 0.08 s	Glint noise correlation time
w	= 1 m	Antenna aperture
τ_{AN}	= 0.6 s	Atmospheric noise correlation time[a]
N'	= 3	Navigation ratio
V_M	= 1500 m/s	Missile velocity
V_T	= 450 m/s	Target velocity (positive for incoming target)
N_T	= 19.6 m/s^2	(2 g) Target maneuver level
T_M	= 2.5 s	Target maneuver time[b]
τ_{MIN}	= 0.5	Minimum achievable system time constant[c]
σ_{ACCLIM}	= 98 m/s^2	(10 g) Maximum allowable missile acceleration caused by noise[d]

[a]Assumed target altitude is below 5 km, hence $R^* = R$.
[b]For a Poisson-distributed step target maneuver with average time between maneuvers T_P, use $T_M = 0.25\ T_P$; here $T_P = 10$ s.
[c]Used only in numerical Examples 2 and 3.
[d]Used only in numerical Example 3.

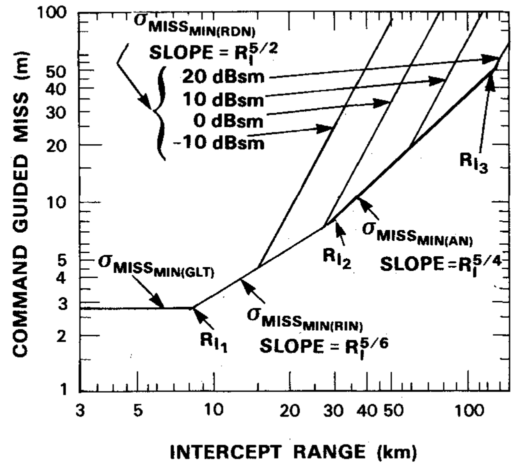

Fig. 4 Asymptotic miss distances of dominant noise and maneuver for Example 1.

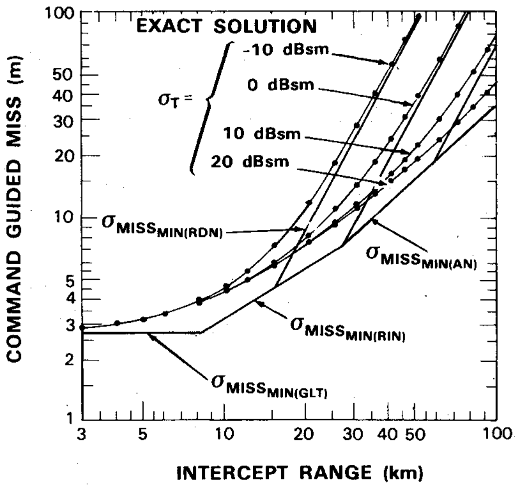

Fig. 5 Dominant noise and maneuver miss distance asymptotes and exact solution for Example 1.

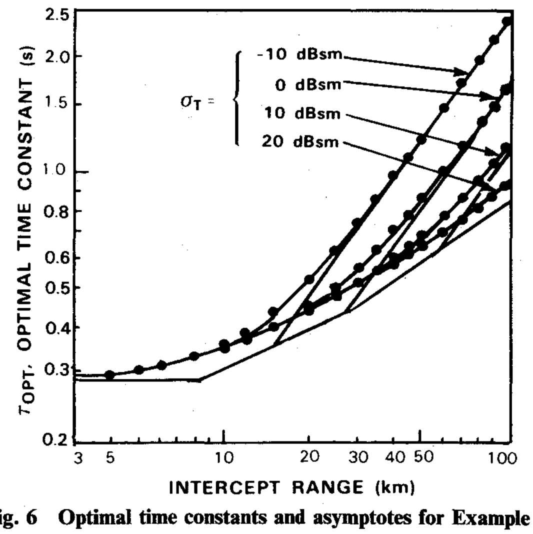

Fig. 6 Optimal time constants and asymptotes for Example 1.

It should be emphasized that these miss distance curves (Figs. 4, 5, 7, and 8) are asymptotes of the minimum rms miss distance that contain maneuver and the largest noise contributor (i.e., glint, range-independent, atmospheric, or range-dependent noise).

Example 1

A numerical example is presented here to illustrate the methodology presented above. Table 4 contains the numerical values of system variables. Figure 4 is the plot of the asymptotic miss distances for the example.

In order to illustrate that these asymptotes for miss distances are good approximations to the exact solutions, the exact solutions are presented in Fig. 5 as curves with dots over a copy of Fig. 4. The exact solution is calculated from Eq. (7). As is seen by this comparison, the agreement between the asymptotes and the exact solution is very good and only departs significantly near breakpoints in the asymptotes.

Figure 6 contains the time constants, τ_{OPT} [see Eq. (10)], corresponding to the exact solutions of Fig. 5; also displayed are the asymptotes of τ_{OPT} calculated from the maneuver spectral density and the dominant noise spectral density.

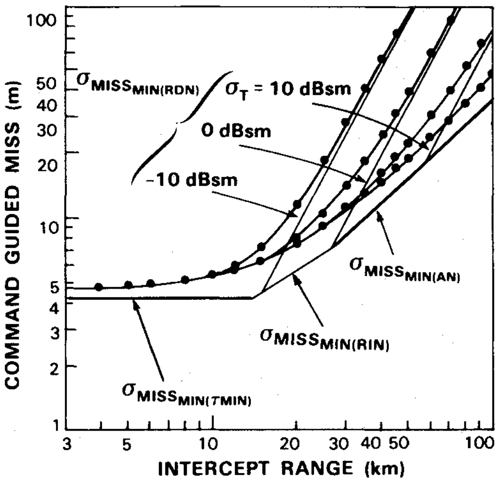

Fig. 7 Noise-maneuver asymptotes with minimum time constant constraint and exact solution, Example 2.

V. Realistic System Limitations

In this section, two realistic limitations are discussed that impact and modify the results of the previous section. They are the minimum achievable total system time constant and maximum achievable missile acceleration.[1] In the considerations in the previous section, it was implicitly assumed that there was no bound on the achievable system time constant nor on the achievable missile acceleration.

Missile System Time Constant Limitation

Missile autopilots and airframes are designed to achieve desired responses, but they become difficult, if not impossible to design as the autopilot-airframe response time constant required becomes very short. Consequently, there is always a lower bound on the achievable system time constant, τ_{MIN}; the optimal time constant τ_{OPT} defined in the previous section can be used only if it exceeds τ_{MIN} at each operating point (i.e., intercept range).

Equation (8), repeated here

$$\sigma^2_{MISS} = \left(\sum \phi\right) K_o \tau^{-1} + \phi_{MVR} K_M \tau^5 \tag{8}$$

shows that for system time constants greater than the optimal time constant, τ_{OPT}, the miss distance is determined mainly by the maneuver miss; consequently the asymptotic miss for the minimum achievable system time constant is approximately

$$\sigma_{MISS_{MIN(\tau MIN)}} \cong (\phi_{MVR} K_M)^{1/2} \tau^{5/2}_{MIN} \tag{19}$$

where the notation MIN(τMIN) indicates that the minimum miss distance is set by the minimum time constant constraint.

Example 2

The asymptotic miss for the minimum achievable system time constant for $\tau_{MIN} = 0.5$ s is plotted in Fig. 7, as Example

Table 5 Normalized steady-state acceleration adjoint coefficients $K_o'(n\tau)$ for a fifth-order system with equally distributed time constants

	N'		
n	3.0	3.5	4.0
1.0	264	368	494
1.5	119	165	220
2.0	68	93	124
2.5	44	60	79
3.0	30	42	55

2; it appears as the horizontal line at the 4.2 m level. This case is the same as Example 1 except that the time constant limitation is included. Exact solutions [Eq. (7)] are represented as curves with dots. There is good agreement between the asymptotes and the exact solutions. Comparison with Fig. 5 indicates that the minimum system time constant limitation increases miss distance only at short ranges.

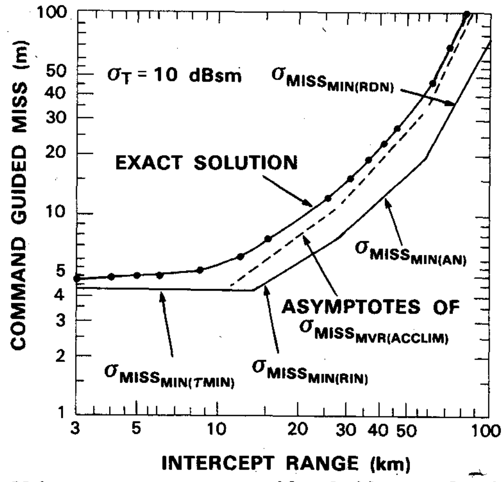

Fig. 8 Noise-maneuver asymptotes with and without acceleration limits and exact solution, Example 3.

Missile Acceleration Limitation

Maximum achievable missile acceleration is another realistic limitation that must be considered. If the guidance system were to command and achieve more acceleration than the design limit of the system, various components would be likely to fail, thereby causing a very large miss distance. If there are internal limits imposed on the acceleration command, and those limits are exceeded by noise-induced acceleration commands, then there is no acceleration capability left to counter target maneuver, resulting in large miss distances. Consequently, it is an important design practice to avoid acceleration saturation.[1,2] In this section, it is shown that acceleration saturation due to noises can be avoided by imposing another minimum time constant constraint that is dependent upon system parameters and intercept range.

The various noises (glint, range-independent, atmospheric, and range-dependent) not only cause miss distance at the end of the flight, according to the equations of Table 3, but they also induce accelerations during the flight. The adjoint technique can also be used to develop equations for the rms accelerations induced by noises.[2] It is accomplished by using the same adjoint system as for the miss distance except that the impulse is applied at the point corresponding to commanded acceleration in the basic forward system diagram; this changes the adjoint results into accelerations instead of miss distances. Following the same development as in the previous section which deals with the steady-state adjoint solutions, equations can be developed for rms accelerations that are similar to the equations of Table 3. A new set of normalized steady-state adjoint coefficients K_n' are found by applying the impulse at Δ, where Δ is the time-to-go before intercept at which the acceleration occurs. These coefficients increase as time-to-go decreases, i.e., as the point of intercept is approached. The other difference is that the equations all have an extra factor of τ^{-4}.

The exact expression for rms commanded acceleration involves polynomials in missile and target velocity and missile system time constant. Once again, the terms involving K_o' are the larger terms for many practical parameter values. As a result, the variance of the commanded acceleration due to noise alone can be approximated as

$$\sigma^2_{\text{ACC}}(t_{\text{go}}) \approx \left(\sum \phi\right) K_o'(t_{\text{go}})\tau^{-5} \tag{20}$$

The commanded acceleration needs to be kept below some specified value only until about one system time constant before intercept because any commands issued after that time would not have much effect. Table 5 contains numerical values of K_o' for t_{go} equal to 1 to 3 system time constants and for the navigation ratios $N' = 3$, 3.5, and 4, for a fifth-order system with equally distributed time constants.

The variance of the noise-induced acceleration command is inversely proportional to the fifth power of the system time constant. Consequently, a minimum system time constant can be specified to keep the missile out of acceleration saturation at the specified time-to-go:

$$\tau_{\text{ACCLIM}} = \left[\left(\sum \phi\right) \frac{K_o'(t_{\text{go}})}{\sigma^2_{\text{ACCLIM}}}\right]^{1/5} \tag{21}$$

where σ_{ACCLIM} = Specified limit on acceleration saturation caused by noises.

This time constant constraint is imposed only if it is larger than the minimum achievable time constant indicated above, τ_{MIN}, and the optimal system time constant, τ_{OPT}, to achieve minimum miss distance. Where the time constant must be constrained by acceleration limits, a situation which will occur for the longer ranges, miss distance will be determined mainly by the maneuver mean square miss distance, which is

$$\sigma^2_{\text{MISS}_{MVR(\text{ACCLIM})}} = \frac{\left(\sum \phi\right) \phi_{MVR} K_M K_o'(t_{\text{go}})}{\sigma^2_{\text{ACCLIM}}} \tag{22}$$

where the notation ACCLIM indicates that the minimum miss is set by the acceleration limit. Consequently, by requiring the missile to remain below the acceleration limit, the noise level determines the maneuver miss which dominates the total miss distance.

Example 3

The curve with dots in Fig. 8 shows the exact adjoint solution for the 10 dBsm target of Example 2 when τ_{ACCLIM} is used as the system time constant whenever

$$\tau_{\text{ACCLIM}} > \tau_{\text{OPT}} \quad \text{and} \quad \tau_{\text{MIN}} \tag{23}$$

for the case where

$$\sigma_{\text{ACCLIM}} = 98 \text{ m/s}^2 \text{ at } t_{\text{go}} = 1\tau \tag{24}$$

i.e., the acceleration command due to noise is limited at one time constant before intercept. The plotted solid line asymptotes are the same as in Fig. 7. The dashed lines are the asymptotes of maneuver rms miss distance of Eq. (22); the exact solution is close to the maximum of the dashed and solid line asymptotes.

At the longest ranges, where range-dependent noise dominates the determination of the acceleration constrained system time constant, rms miss distance increases as the third power of intercept range. This range dependence is stronger than the five-halves power dependence noted in the non-acceleration-limited analysis. Consequently, the miss distances are larger than the non-acceleration-limited noise asymptotes at long range.

Correlated Noise Effects

As indicated at the beginning of this paper, both the atmospheric and glint noises are correlated noises. Correlated noise is usually treated in the adjoint technique by modeling it as white noise passed through a low-pass filter.[8,9] If the bandwidth of the system that the noise is entering is narrower than the bandwidth of the correlated noise, the noise still appears white to the system, and the filter can be eliminated in the modeling. This was the case for the glint noise assumed in the examples.

If the bandwidth of the noise is narrower than the bandwidth of the system, as in the case of atmospheric noise, then the effect of the filter is important; it reduces the total noise energy entering the system, in a way that depends upon the bandwidths of the noise and the system. In order to evaluate this effect for atmospheric noise, the problem was solved with (and without) the low-pass filter to represent the correlation at various solution points using the full dynamic adjoint solution.[3] The effect of the inclusion of the filter was to reduce the variance of the miss distance caused by atmospheric noise to about 40% of the value obtained when the filter was not used. This occurred over a variety of conditions of interest. Therefore, the effect of atmospheric noise correlation was approximated by multiplying the spectral density of the atmospheric noise by a correlated noise coefficient, f_c, of 0.4.

VI. Summary and Conclusions

Equations for the statistical miss distances caused by various noise sources and target maneuver have been extended to the command guidance system from the homing guidance steady-state adjoint solutions; they are all analytical functions of the total system time constant.

An optimal time constant was derived, using a simplifying assumption, that minimizes the total statistical miss distance. The minimum achievable system time constant was considered as a constraint. Based upon the optimal achievable system time constant, curves of miss distance vs range-to-intercept were developed which depend upon expected target maneuver level, level of servo noise, target-radar cross-section, glint noise level, and atmospheric noise level. The optimal time constant to achieve minimum statistical miss distance is a result of balancing miss distance caused by target maneuver with miss distance caused by noises; consequently, equations for the asymptotes to these curves were developed by considering only the noise that dominates in each intercept range. This easily drawn set of asymptotes is very useful in understanding the limitations on system performance at the different intercept ranges and the sensitivities of the results to the parameter values used in the analysis. These asymptotes show that for a typical system at the short ranges either the glint noise or the minimum achievable system time constant is the limiting factor that prevents the miss distance from reaching zero; at longer ranges, the servo noise dominates; at even longer ranges, the atmospheric noise is the dominant noise; only at the longest ranges is it the range-dependent noise that limits performance. The uncertainty of a priori knowledge of target-radar cross-section is handled by creating a set of radar-cross-section-dependent asymptotes in the region where range-dependent receiver noise dominates. These asymptotes are shown to be good approximations to the exact solution.

Finally, another important realistic lower constraint on achievable time constant was imposed to avoid missile acceleration saturation near the end of the flight which would cause huge miss distances. Inclusion of this constraint yields larger miss distances only at the longer ranges.

This asymptotic technique should be very helpful to the system analyst in developing a systematic understanding of the performance limitations of command guidance missile systems. The exact equations for statistical miss distance are useful in the performance analysis of command guided systems.

The equations derived in this paper for the optimal system time constant are valuable to the system designer for achieving minimum miss distance over the ranges of system parameters and limitations, and intercept conditions; further, these equations can be evaluated to determine the zone where the less complex and less costly command guidance missile system can be effective.

Acknowledgments

This work was sponsored by the U.S. Air Force under contract F19628-85-C-0002. The views expressed are those of the author and do not reflect the official policy or position of the U.S. Government. The author is especially grateful to Dennis Keane for his insights, suggestions, clarifications of equations, editing, encouragement, and enthusiasm in pursuing and developing this topic. The encouragement and technical suggestions of Dr. F. William Nesline and Paul Travers were vital to this development. Special thanks are due to Michael Mullins, who programmed the technique, reviewed the derivations, performed numerous cross-checks on the computer, and plotted results, and to William Alpert for his editorial assistance.

References

[1]Nesline, F. W., Jr. and Nesline, M. L., "An Analysis of Optimal Command Guidance vs Optimal Semi-Active Homing Missile Guidance," American Control Conference, Seattle, WA, June 18–20, 1986.

[2]Nesline, F. W., Jr. and Zarchan, P., "Miss Distance Dynamics in Homing Missiles," AIAA Paper 84-1845, Aug. 1984.

[3]Zarchan, P., "Complete Statistical Analysis of Nonlinear Missile Guidance Systems — SLAM," *AIAA Journal of Guidance and Control*, Vol. 2, Jan. 1979.

[4]Howe, R. M., *System Engineering Handbook*, edited by R. E. Machol, McGraw Hill, New York, 1965, Chap. 19.

[5]Laning, J. H. and Battin, R. H., *Random Processes in Automatic Control*, McGraw-Hill, New York, 1956, Chap. 6 and Appendix F.

[6]Peterson, E. L., *Statistical Analysis and Optimization of Systems*, Wiley, New York, 1961, Chaps. 4 and 6.

[7]"The Method of Adjoint System and Its Application to Guided Missile Noise Studies," EASAM, Camberley, England, NTIS Rept. PB186191, 1969.

[8]Fitzgerald, R. J. and Zarchan, P., "Shaping Filters for Randomly Initiated Target Maneuvers," *Proceedings of the AIAA Guidance and Control Conference*, AIAA New York, Aug. 1978, pp. 424–430.

[9]Schwartz, M., *Information Transmission, Modulation, and Noise*, McGraw-Hill, New York, 1959, pp. 441–448.

[10]Barton, D. K. and Ward, H. R., *Handbook of Radar Measurement*, Prentice-Hall, Inc., Englewood Cliffs, NJ, 1969, p. 380.